Mastering a financial calculator

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Using this manual

If you are using this manual, chances are that you are currently in a financial planning certificate program, considering taking the CFP® Exam, or currently studying for the CFP® Exam. The calculations covered are content that could appear on the exam.

This manual is written for both Hewlett Packard (**HP) 10B (**10BII or 10BII+) financial calculator users as well as **HP 12C** (or 12C Platinum) financial calculator users. The basic principles and terminology related to financial calculations (e.g., time value of money) are consistent across both calculators. Going forward, these calculators will be referred to as simply **10B** and **12C**.

Introduction to the HP 10B and HP 12C

Using multiple functions for keys

10B users

HP 10B calculators have alternate functions shown with gold text. When you need to enable these functions, the manual will indicate **gold** as the key to press for the desired function. There are also purple functions that can be accessed on the 10B; however, they aren't used for the calculations discussed in this manual. On more recent versions of the 10B, the "gold" key is more orange in color, but this manual will refer to it as **gold**.

12C users

Most of the keys on the 12C have multiple functions. Keys have **white** text on the key, **blue** text below the white, and **orange** text above the key.

- To enable the function of the white text, simply press the key
- To enable the function of the blue text, press the blue **g** key and then the desired key
- To enable the function of the orange text, press the orange **f** key and then the desired key

For the remainder of this manual, these keys will be referred to as **f** and **g**.

Turning on the calculator

10B users

On the HP 10B, the **ON** key turns the calculator on. To turn the 10B off, press **gold** followed by **ON**.

12C users

Activate the HP 12C by pressing the **ON** key. If you accidentally leave it on, the HP 12C will shut itself off automatically after a short amount of time. Press the **ON** key again to turn off the calculator. When you turn your calculator back on, the display will show the last number that was displayed before it turned itself off. So, unlike traditional calculators, turning off the calculator does not clear the display (see *Clearing the display and registries* below to learn how to clear the display).

Initial set up: 10B

Setting the number of decimal places
Although most financial planning problems (presented in certification exams) are solved using two decimal places, certain kinds of problems may involve three or even four decimal places. To ensure you are ready for any problem, we suggest you leave your calculator set to 4 decimal places (you can always round yourself if you need fewer).

To set the 10B to 4 decimal places, press **gold** then the **DISP** key (the key with = in white), followed by the **4** key. Your display should now show 0.0000.

Clearing display and registries
To completely clear the 10B, press the **gold** key followed by the **C ALL** key (the key with C in white) For the rest of the manual, this key will be referred to as **clear all**.

This clears the display as well as anything stored in the registries (storage is not used in this manual). **It is important that you get into the habit of clearing your calculator before you begin every calculation.** If you do not clear out the registries between problems, subsequent problems may use data from old problems and could provide the wrong answer. For this reason, throughout this manual, you will be reminded to clear your calculator prior to beginning each problem.

Initial set up: 12C

Setting the number of decimal places
Although most financial planning problems (presented in certification exams) are solved using two decimal places, certain kinds of problems may involve three or even four decimal places. To ensure you are ready for any problem, we suggest you leave your calculator set to 4 decimal places (you can always round yourself if you need fewer).

To set the 12C to display 4 decimal places, press the **f** key followed by the **4** key (you can change your display to a different number of decimal places in the future by pressing the **f** key and then the key corresponding to the number of decimal places you desire).

Clearing display and registries
To completely clear the 12C before beginning any operation, press the **f** key followed by the **CLX** key.

This clears the display as well as anything stored in the registries (storage is not used in this manual). **It is important that you get into the habit of clearing your calculator before you begin every calculation.** If you do not clear out the registries between problems, subsequent problems may use data from old problems and could provide the wrong answer. For this reason, throughout this manual, you will be reminded to clear your calculator prior to beginning each problem.

Doing Basic Math on the 10B and 12C

Basic math on the 10B

Basic math calculation on HP 10B calculators is similar to a traditional calculator. If you are a 10B user, you may skip to the ***More complex math example*** section below.

Basic math on the 12C

Use of Reverse Polish Notation (RPN)
Unlike a traditional calculator, the HP 12C uses Reverse Polish Notation (RPN) for basic math (addition, subtraction, multiplication, and division) functions. Using the HP 12C for basic calculations takes a little getting used to and practice; therefore, while you're trying to get the hang of the HP 12C, it's a good idea to put your traditional pocket calculators in a drawer for a month or two and use the financial function calculator exclusively.

If you happen to have a **HP 12C Platinum**, you can switch from RPN to ALG (algebraic) to get the feeling of a traditional calculator (with the exception that the ENTER key takes the place of the = key). To change the HP 12C Platinum to algebraic mode, press the **f** key followed by the **ALG** key (the key with **EEX** in white—more about using the orange and blue keys below). The display will change from RPN to ALG. If you wish to change the mode back to RPN, press **f** followed by the **RPN** key (the key with **CHS** in white). The display will change from ALG to RPN.

Note that this manual always uses RPN for 12C keystrokes.

12C basic math example

As mentioned above, the HP 12C uses RPN for basic math operations. The answer to a calculation automatically appears after the operation key (+, –, ×, ÷) is pressed at the end of the calculation. As a result, there is no equal sign on the 12C. Because the operation comes after the numbers, you will need to press the **ENTER** key between numbers to let the calculator know you are entering another number.

Example

Let's say you want to use your 12C to determine the annual amount of mortgage payments. You know that the monthly payments are $799. Obviously, the annual amount will be $799 times 12 months. With a traditional calculator (or a 10B), you would simply enter 799 × 12 =. With the 12C, the operator (×) goes at the end and the numbers are separated by the **ENTER** key. Don't forget to begin by clearing your calculator first. See keystrokes for the 12C and 10B (or traditional calculator) below.

10B	12C
gold, clear all	f, CLX
799	799
×	ENTER
12	12
=	×

Answer: 9,588

More complex math example

For more complex calculations such as multiplying or dividing the sum or difference of other calculations on the 12C, complete the calculations in parentheses first and then enter the × or ÷ at the end. For the same calculations on the 10B, you can use parentheses to separate the interim calculations. The open parenthesis is accessed by pressing **gold** then the **(key** (the key with RM in white), and the close parenthesis is accessed by pressing **gold** then the **)** key (the key with M+ in white). See the example below for keystrokes to help this make more sense.

Example

(43 + 19) × (64 – 47) would be entered as follows:

10B	12C
gold, clear all	f, CLX
gold, (	43
43	ENTER
+	19
19	+
gold,)	64
×	ENTER
gold, (	47
64	–
–	×
47	
gold,), =	

Answer: 1,054

A word on rounding off numbers

When working with dollar amounts, it is an accepted practice to round off to the nearest dollar. Fractions of dollars of $0.50 or higher should be rounded up to the next whole dollar. Fractions of dollars which are lower than $0.50 ($0.01 to $0.49) should be rounded down to the next whole dollar. Do not round off percentages or calculations steps. Round off final answers only.

Practice Problems

1. 156 + 1,952

10B	**12C**
gold, clear all	f, CLX
156	156
+	ENTER
1952	1952
=	+

Answer: 2,108

2. 9.87 – 2.35

10B	**12C**
gold, clear all	f, CLX
9.87	9.87
–	ENTER
2.35	2.35
=	–

Answer: 7.52

3. 2,180 × 6.7

10B	**12C**
gold, clear all	f, CLX
2180	2180
×	ENTER
6.7	6.7
=	×

Answer: 14,606.

4. 541 ÷ 33

10B	**12C**
gold, clear all	f, CLX
541	541
÷	ENTER
33	33
=	÷

Answer:16.39

5. (10 + 82) × (66 – 18)

10B	**12C**
gold, clear all	f, CLX
gold, (	10
10	ENTER
+	82
82	+
gold,)	66
×	ENTER
gold, (	18

66	–
–	×
18	
gold,)	
=	

Answer: 4,416

Time value of money calculations

The concept of Time Value of Money Theory

The Time Value of Money Theory is best expressed in a single sentence: *A dollar you receive today is worth more than a dollar you receive tomorrow*. And conversely: *A dollar you receive tomorrow is worth less than a dollar you receive today.*

The reason behind this theory is that once money is in your hands, it can be immediately used for investing to make more money. Conversely, while another party is still holding your money, you lose the opportunity to have that money working for you immediately. The effect of the time value of money is not the same as the effect of inflation. The Time Value of Money Theory applies whether there is inflation, deflation, or disinflation. The Time Value of Money Theory focuses on the opportunity to use money.

Elements in a time value of money calculation

In every time value of money problem, you will be given at least three of the five elements listed below. Numbers may be entered as either positive or negative values (explained in detail in the *Positive input* and *Negative input* sections below). Each of the elements below has a corresponding key on the top row of both the 12C and 10B.

Element	**Key**	**Definition**
Present Value*	**PV**	The beginning value of an investment, even if the scenario is set in the future. For instance, "Claire retired this year with investments of $500,000 from the account she opened with $25,000 thirty years ago." The present value is negative $25,000*.
Future Value	**FV**	The ending value of an investment. In the example above, the future value is $500,000 even though it is the value of Claire's account today.
Payments	**PMT**	A periodic amount either added to (negative value) or drawn from (positive value) an investment.
Number of periods	**N** (10B) or **n** (12C)	The number of compounding or discounting periods.
Interest	**I/YR** (10B) or **i** (12C)	The interest earned or discount rate of an investment.

*Present value in time value of money problems is nearly always a negative value (more on this below).

Using positive and negative values

Positive input
Money is *deposited* into the client's checking account*. Examples include the following:

- CD or bond interest received by the client
- Dividends received by the client
- Pension or IRA distributions received by the client, but not pension deposits
- Withdrawals from mutual funds received by the client
- Rent received from an investment property owned by the client

Negative input
Money is *withdrawn* from the client's checking account*. Examples include the following:

- Investments into mutual funds, stocks, bonds, IRAs
- A repair is made to the investment property owned by the client
- Pension or 401(k) **deposits** made by client's employer. The employer must write the check (negative), but the money isn't deposited into the client's personal account until he takes distributions (positive). If you enter the pension deposit or 401(k) deposit as a positive, you will get a wrong answer or no answer.

*A checking account does not operate as an investment. It is a holding account for money. When your client writes a check, there is a negative entry. When your client receives a check, there is a positive entry. Obviously, today, many transfers are completed electronically, but the concept is the same: funds transferred out are negative and funds transferred in are positive.

To toggle a value between positive and negative on the 10B, enter the value followed by the **+/–** key. To toggle a value between positive and negative on the 12C, enter the value followed by the **CHS** key (CHS = change sign).

Use of Payment BEGIN or END mode

Using Begin or End mode determines when the payment was made in the period. Both the 10B and 12C default to End mode. To toggle between Begin and End on the 10B, press the **gold** key followed by the **Beg/End** key (the key with **MAR** in white). BEG will appear on the display in Begin mode and disappear in End mode.

To change to Begin mode on the 12C, press **g** followed by **BEG** (the key with **7** in white). BEGIN will appear on the display. To change to End mode, press **g** followed by **END** (the key with **8** in white). BEGIN will disappear when the calculator is in End mode.

If the calculator shows "No Solution" or "Error 5" you likely used the wrong sign (+/-) when entering a number.

You will not need to worry about which mode your calculator is in if the problem does not deal with periodic payments.

The following are situations where Begin and End modes are used:

Begin mode

- College tuition paid
- Retirement benefits received
- Family needs
- Problems that explicitly mention payments at the beginning of the period

End mode

- 401(k) deferrals
- Profit-sharing contributions
- Bond interest paid
- Mortgage payments

Solving time value of money problems

Many problem solvers like to place their problem elements in a box because it helps them to visualize the solution process:

Present Value	
Future Value	
Payments	
Number of periods	
Interest	

Example

Today, Dave Betterman deposited $10,000 into a savings account earning 5% compounded annually. What will the balance be in 10 years?

If you were solving this problem, you would start by filling in the time value elements box as follows:

Present Value	–10000
Future Value	solve for
Payments	N/A
Number of periods	10
Interest	5

Notice that the present value is negative because funds will be moving *from* Dave's checking account to his savings account.

Determining Future Value

To determine the future value, you will need to know the present value, number of periods, and rate of return (also known as interest).

Example

Jane and Jeff Peters just bought a cabin overlooking Lake Shmata for $36,000. They expect the cabin to appreciate at an average rate of 8% per year. If they decide to sell the cabin and move to Florida when Jane retires in 11 years, what should the cabin be worth at that time?

First, fill in your time value elements box:

Present Value	–36000
Future Value	solve for
Payments	N/A
Number of periods	11
Interest	8

Then, enter them into your calculator using the following keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
36000, +/–, PV	36000, CHS, PV
11 N	11 n
8 I/YR	8 i
FV	FV

Answer: $83,939

Notice that we made the value for the initial investment of $36,000 *negative* to indicate that funds are moving *out* of our checking account to pay for the cabin.

Time Value of Money problems when interest compounds multiple times per year

In the example of the Peters 'cabin above, the investment appreciated annually. When an investment compounds more than annually (i.e., semiannually, quarterly, monthly, or daily), the interest rate must also be adjusted by the same amount. You must place both the interest rate and the number of periods into the same fractional-year parts. The interest element is always **divided** by the number of periods within each year. The number of periods (n) element is always **multiplied** by the number of periods within each year.

Dealing with compounding periods on the 10B
Before beginning a time value of money problem, you will need to input the number of compounding periods per year followed by the **gold** key and then the **P/YR** key (the key with **PMT** in white). So, for a problem with quarterly compounding, the keystrokes would be **4**, **gold, P/YR**. The number of payments per year will stay in the calculator until it is changed. To see what the current setting is, press **gold** then **clear all**. The number of payments per year will appear briefly on the display.

To be safe, it's a good idea to reset the number of payments per year before you begin a time value of money calculation, and this manual includes these keystrokes.

Because you will enter the number of compounding periods per year at the beginning of your calculations, you do not need to worry about dividing the interest by the number of periods later; however, you will also need to multiply the number of years by the number of compounding periods per year to get the correct answer. To do this, enter the number of years, and then press the **gold** key followed by the **×P/YR** key (the key with N in white).

Dealing with compounding periods on the 12C
There isn't a periods per year function on 12C calculators, so you will need to do the math separately when solving problems in which interest compounds more frequently than annually.

If you are having difficulty remembering which needs to be divided and which needs to be multiplied, you can look at the n and i keys on your 12C. You'll notice that the **n** key has **12×** in blue and the **i** key has **12÷** in blue. This is the reminder to multiply the number of periods and divide the interest rate.

Example

Max placed $25,000 in a money market deposit account which is expected to provide an average return of 7% interest compounded annually. What will the balance be in 9 years?

Elements:

Present Value	–25000
Future Value	solve for
Payments	N/A
Number of periods	9
Interest	7

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
25000, +/–, PV	25000 CHS PV
9 N	9 n
7 I/YR	7 i
FV	FV

Answer: $45,961

Now, let's look at the same example except with the interest compounding semi-annually (twice per year):

Elements:

Present Value	–25000
Future Value	solve for
Payments	N/A
Number of periods	9 × 2
Interest	7 (÷ 2 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
2, gold, P/YR	---
25000, +/–, PV	25000 CHS PV
9, gold, ×P/YR	9, ENTER, 2, ×, n
7 I/YR	7, ENTER, 2, ÷, i
FV	FV

Answer: $46,437

Notice that the amount is larger than the one you determined using annual compounding only. This is an illustration of the power of compounding interest. Each time the interest compounds, the current amount of the investment becomes higher; therefore, the next time the interest compounds on this higher amount it will yield a higher value.

Practice Problems

1. John Bailey deposited $23,000 into a money market account paying 8.5% interest compounded daily (using a 360-day year). How much will the money market hold in 3 years?

Elements:

Present Value	–23000
Future Value	solve for
Payments	N/A
Number of periods	3 × 360
Interest	8.5 (÷ 360 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
360, gold, P/YR	---
23000, +/–, PV	23000, CHS, PV
3, gold, ×P/YR	3, ENTER, 360, ×, n
8.5, I/YR	8.5, ENTER, 360 ÷, i
FV	FV

Answer: $29,680

2. Luis Carmel purchased a lithograph for $3,700 eight years ago. He estimates the average annual rate of return on the artwork has been 12%. Approximately how much is the picture worth today?

Elements:

Present Value	–3700
Future Value	solve for
Payments	N/A
Number of periods	8
Interest	12

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
3700, +/–, PV	3700, CHS, PV
8, N	8, n
12, I/YR	12, i
FV	FV

Answer: $9,161

3. Mary Wong invested $15,000 into a CD paying 7.75% interest compounded quarterly 9 years ago. How much is Mary's CD worth today?

Elements:

Present Value	–15000
Future Value	solve for
Payments	N/A
Number of periods	9 × 4
Interest	7.75 (÷ 4 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
4, gold, P/YR	---
15000, +/–, PV	15000, CHS, PV
7.75, I/YR	7.75, ENTER, 4, ÷, i
9, gold, ×P/YR	9, ENTER, 4, ×, n
FV	FV

Answer: $29,931

4. Kris Adams would like to have $50,000 in 3 years for a down payment on a new house. If she deposits $37,000 today into an investment vehicle paying 12% compounded semi-annually, will she have the money for her down payment in 3 years?

Elements:

Present Value	–37000
Future Value	solve for
Payments	N/A
Number of periods	3 × 2
Interest	12 (÷ 2 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
2, gold, P/YR	---
37000, +/–, PV	37000, CHS, PV
3, gold, ×P/YR	3, ENTER, 2, × n
12, I/YR	12, ENTER, 2, ÷, i
FV	FV

Answer: Yes, Kris will have $52,485 in three years.

5. Today is Herman Tippin's favorite nephew, Sam's, second birthday. In honor of the occasion, Herman has just deposited $20,200 into a savings account for Sam. The account will pay 5.25% interest compounded monthly. What will the account balance be on Sam's 19th birthday?

Elements:

Present Value	–20200
Future Value	solve for
Payments	N/A
Number of periods	17 × 12
Interest	5.25 (÷ 12 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
12, gold, P/YR	---
20200, +/–, PV	20200, CHS, PV
17, gold, ×P/YR	17, ENTER, 12, ×, n
5.25, I/YR	5.25, ENTER, 12, ÷, i
FV	FV

Answer: $49,217

Determining Present Value

As mentioned above, in every time value of money problem, you will be given at least three of the five elements. If you are provided with the future value (FV), the number of compounding periods (n or N), and the interest rate (i or I/YR), you can solve for a present value. Remember that the present value is the amount that began an investment sequence. In present value problems, interest doesn't compound forward; it discounts backwards in time. The applicable rate is called the discount rate.

Example

Olga Maxwell closed her passbook savings account today. She received $30,000. Olga opened the account 4 years ago and hasn't made any other deposits. If she has been earning an annual interest rate of 5.5% on her money, how much did Olga originally invest?

Elements:

Present Value	solve for
Future Value	30000
Payments	N/A
Number of periods	4
Interest	5.5

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
30000, FV	30000, FV
4, N	4, n
5.5, I/YR	5.5, i
PV	PV

Answer: –$24,217

Please Note: The answer is negative. This is because the sequence began with a deposit which is a negative outflow. The CFP® Exams will not provide negative amounts as solutions to present value problems; however, it is important that you understand why the negative result on your calculator is correct.

Practice Problems

1. Geraldine Chase inherited a diamond necklace from her grandmother 4 years ago. It was just appraised at $45,000. What was the value of the necklace when Geraldine inherited it if its value has appreciated at a 5% average annual rate?

Elements:

Present Value	solve for
Future Value	45000
Payments	N/A
Number of periods	4
Interest	5

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
45000 FV	45000, FV
4 N	4, n
5 I/YR	5, i
PV	PV

Answer: $37,022

2. Pete Palukas expects to inherit $98,000 from a trust fund in 13 years. What is the value of the trust now if it is discounted 9.25% semi-annually?

Elements:

Present Value	solve for
Future Value	98000
Payments	N/A
Number of periods	13 × 2
Interest	9.25 (÷ 2 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
2, gold, P/YR	---
98000, FV	98000, FV
13, gold, ×P/YR	13, ENTER, 2, ×, n
9.25, I/YR	9.25, ENTER, 2, ÷, i
PV	PV

Answer: $30,248

3. Martha Jones is starting college today. Her father started saving for her education 16 years ago and now has $60,000 ready to pay for her expenses. How much did Mr. Jones originally deposit if the interest rate has been 8% compounded quarterly?

Elements:

Present Value	solve for
Future Value	60000
Payments	N/A
Number of periods	16 × 4
Interest	8 (÷ 4 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
4, gold, P/YR	---
60000, FV	60000, FV
16, gold, ×P/YR	16, ENTER, 4, ×, n
8, I/YR	8, ENTER, 4, ÷, i
PV	PV

Answer: $16,894

4. David Chen invested in a 5-year vehicle which will be worth $15,000 at maturity. He will receive 7.5% interest compounded monthly. How much did David deposit to fund this investment?

Elements:

Present Value	solve for
Future Value	15000
Payments	N/A
Number of periods	5 × 12
Interest	7.5 (÷ 12 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
12, gold, P/YR	---
15000 FV	15000, FV
5, gold, ×P/YR	5, ENTER, 12, ×, n
7.5, I/YR	7.5, ENTER, 12, ÷, i
PV	PV

Answer: $10,321

5. Dawn Riggs owns 5 gold coins valued at $682 apiece. If the coins have appreciated 5% every year for the last 9 years, how much was each coin worth 9 years ago?

Elements:

Present Value	solve for
Future Value	682
Payments	N/A
Number of periods	9
Interest	5

Keystrokes:

10B	12C
gold, clear all	f, CLX
1, gold, P/YR	---
682, FV	682, FV
9, N	9, n
5, I/YR	5, i
PV	PV

Answer: $440

Determining the interest rate of an investment

If you know the present value (PV), the future value (FV), and the number of compounding or discounting periods (n or N), you have the elements you need to determine the correct interest or return rate (i or I/YR).

Example

Roger Webster would like to open an auto repair shop in 10 years. He has $40,000 now and estimates that he will need $100,000 to get the business started. What rate of return would Roger require in order to have the funds in time to reach his goal?

Elements:

Present Value	–40000
Future Value	100000
Payments	N/A
Number of periods	10
Interest	solve for

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
40000, +/–, PV	40000, CHS, PV
100000, FV	100000, FV
10, N	10, n
I/YR	i

Answer: 9.6%

Determining the interest rate when interest compounds multiple times per year

When there are multiple compounding periods per year, the 12C will return the interest rate for that period only, therefore; 12C users will need to multiply the result by the number of compounding periods.

Because 10B users will enter the number of periods per year at the beginning of the calculation, and the calculator's button is I/YR (interest per year), the result is for the interest rate per year and doesn't need to be multiplied by the number of compounding periods.

Remember that number of periods is the number of years multiplied by the number of compounding periods. On the 12C, this math is done before entering the **n** key. On the 10B, this is accomplished by entering the number of years followed by the **gold** key and the **×PR/YR** key (the key with N in white).

Example

What if in the example above, Roger's investment compounded quarterly?

Elements:

Present Value	–40000
Future Value	100000
Payments	N/A
Number of periods	10 × 4
Interest	solve for (×4 for 12C)

Keystrokes:

10B	12C
gold, clear all	f, CLX
4, gold, P/YR	---
40000, +/–, PV	40000, CHS, PV
100000, FV	100000, FV
10, gold, ×P/YR	10, ENTER, 4, ×, n
I/YR	i, 4, ×

Answer: 9.27%

Notice that a lower rate of return is required when interest compounds more frequently within the period.

Practice Problems

1. Adelaide Morton purchased 60 shares of RDX industries for $19.00 per share 14 years ago. Today, she sold 30 shares for a total of $1,617. What was the average annual compound rate of return before taxes on the 30 shares Adelaide sold assuming this stock does not pay dividends?

Elements:

Present Value	–19 × 30
Future Value	1617
Payments	N/A
Number of periods	14
Interest	solve for

Keystrokes:

10B	12C
gold, clear all	f, CLX
1, gold, P/YR	---
19, ×, 30, =, +/–, PV	19, ENTER, 30, ×, CHS, PV

1617, FV	1617, FV
14, N	14, n
I/YR	i

Answer: 7.73%

2. Mr. And Mrs. Clay borrowed $12,000 from Mr. Clay's Aunt Josephine to remodel their kitchen. They paid her $16,000 back at the end of 5 years. What was the annual interest rate if the interest was compounded monthly?

Elements:

Present Value	–12000
Future Value	16000
Payments	N/A
Number of periods	5
Interest	solve for (× 12 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
12, gold, P/YR	---
12000, +/–, PV	12000, CHS, PV
16000, FV	16000, FV
5, gold, ×P/YR	5, ENTER, 12, ×, n
I/YR	i, ENTER, 12, ×

Answer: 5.77%.

3. Marla Hernandez invested $20,000 in a CD. In 5 years when the CD matures, she will receive $30,000. If the interest is compounded weekly, what is the annual compound rate of return on the CD?

Elements:

Present Value	–20000
Future Value	30000
Payments	N/A
Number of periods	5 × 52
Interest	solve for

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
52, gold, P/YR	---
20000, +/–, PV	20000, CHS, PV
30000 FV	30000 FV
5, gold, ×P/YR	5, ENTER, 52, ×, n
I/YR	i, 52, ×

Answer: 8.12%.

4. Andre Devine invested $5,800 for his 2-year-old son to use for the child's education. Andre estimates he will need $47,000 by the time the boy turns 17 years old. If the investment Andre has chosen pays interest quarterly, what average annual rate of return will be needed to achieve this goal?

Elements:

Present Value	–5800
Future Value	47000
Payments	N/A
Number of periods	15 × 4
Interest	solve for

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
4, gold, P/YR	---
5800, +/–, PV	5800, CHS, PV
47000, FV	47000, FV
15, gold, ×P/YR	15, ENTER, 4, ×, n
I/YR	i, 4, ×

Answer: 14.19%

5. What annual interest rate must be obtained to triple an investment in 12 years with daily compounding? (Assume a 360-day year)

Elements:

Present Value	–1000
Future Value	3000
Payments	N/A
Number of periods	12 × 360
Interest	solve for

Note that because we weren't given exact numbers, we can pick any number for PV and triple that number for FV to determine the interest rate required.

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
360, gold, P/YR	---
1000, +/–, PV	1000, CHS, PV
3000, FV	3000, FV
12, gold, ×P/YR	12, ENTER, 360, ×, n
I/YR	i, 360, ×

Answer: 9.16%

Determining the Number of Periods

If a problem provides you with the future value (FV), the present value (PV), and the interest rate (i or I/YR), you and your calculator have all the tools for determining the correct number of compounding periods.

Example

The investment salesperson said that at the quarterly compounded 5% rate of return offered by this investment; your client would have no trouble doubling his money. How long would it take for your client's money to double?

Elements:

Present Value	–1
Future Value	2
Payments	N/A
Number of periods	solve for
Interest	5 (÷ 4 for 12C)

Because numbers were not provided for present and future value, we can select any number for present value and double it for future value.

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
4, gold, P/YR	---
1, +/–, PV	1, CHS, PV
2, FV	2, FV
5, I/YR	5, ENTER, 4, ÷, i
N, ÷, 4, =	n, 4, ÷

Answer: 14 years

Notice that the answer you get after you press the N or n key is 55.7976 (10B) or 56 (12C). This is in quarters (because we entered 4 periods per year on the 10B and divided the interest amount by 4 on the 12C); therefore, we need to divide the result by 4 to determine the number of years.

Practice Problems

1. Matthew Martinson purchased a parcel of land for $3,650 last year. The agent who sold him the property assured him that the lot would appreciate at an average annual rate of 11%. Matthew would like to sell the property for $10,000. How long must he own the property to receive $10,000 when he sells if the real estate agent was correct?

Elements:

Present Value	–3650
Future Value	10000
Payments	N/A
Number of periods	solve for
Interest	11

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
3650, +/–, PV	3650, CHS, PV
10000, FV	10000, FV
11, I/YR	11, i
N	n

Answer: 9.66 years (10B)/10 years (12C)

2. Helga Johnson wishes to accumulate $100,000 to open a crafts store. She has $21,000 to invest now. If the investment she chooses pays 9.25% interest compounded semi-annually, how long will it take before Helga reaches her goal?

Elements:

Present Value	–21000
Future Value	100000
Payments	N/A
Number of periods	solve for
Interest	9.25 (÷ 2 for 12C)

Keystrokes

10B	12C
gold, clear all	f, CLX
2, gold, P/YR	---
21000, +/–, PV	21000, CHS, PV
100000, FV	100000, FV
9.25, I/YR	9.25, ENTER, 2, ÷, i
N, ÷, 2, =	n, ENTER, 2, ÷

Answer: 17 years, 6 months (12C), or 17 years, 3 months (10B)

3. Jenny borrowed $6,000 from her parents and has agreed to pay them back $7,500 in one lump-sum. At an interest rate of 3.5% compounded quarterly, how long will it take Jenny to pay back the loan?

Elements:

Present Value	–6000
Future Value	7,500
Payments	N/A
Number of periods	solve for
Interest	3.5 (÷ 4 for 12C)

Keystrokes:

10B	12C
gold, clear all	f, CLX
4, gold, P/YR	---
6000, +/–, PV	6000, CHS, PV
7500, FV	7500, FV
3.5, I/YR	3.5, ENTER, 4, ÷, i
N, ÷, 4, =	n, ENTER, 4, ÷

Answer: 6 years, 6 months (12C) or 6 years, 5 months (10B)

4. How many years will it take for $1,000 to quadruple at a rate of 6% compounded monthly?

Elements:

Present Value	–1000
Future Value	4000
Payments	N/A
Number of periods	solve for
Interest	6 (÷ 12 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
12, gold, P/YR	---
1000, +/–, PV	1000, CHS, PV
4000, FV	4000, FV
6, I/YR	6, ENTER, 12, ÷, i
N, ÷, 12, =	n, ENTER, 12, ÷

Answer: 23 years, 2 months

5. Today, Harlan Applebaum deposited $7,200 into an account earning 12% compounded monthly. Assuming he makes no additional deposits or withdrawals, how long will it take for Harlan to accumulate $29,000?

Elements:

Present Value	–7200
Future Value	29000
Payments	N/A
Number of periods	solve for
Interest	12 (÷ 12 for 12C)

Keystrokes:

10B	12C
gold, clear all	f, CLX
12, gold, P/YR	---
7200, +/–, PV	7200, CHS, PV
29000, FV	29000, FV
12, I/YR	12, ENTER, 12, ÷, i
N, ÷, 12, =	n, ENTER, 12, ÷

Answer: 11 years, 9 months (12C) or 11 years, 8 months (10B)

Determining Payment Amounts

Some economic scenarios do not begin or end with a lump-sum deposit or withdrawal. Rather, there is more than one funding deposit of withdrawal occurrence. Any withdrawals or deposits that are not made in a lump-sum is called a payment (PMT). Your calculator has a PMT key so that you can command it to determine a payment amount. For problems where the payments are made toward an investment, the investment goal is the future value (FV).

Negative and positive values apply as they did with present value: if the periodic payment goes *from* the investor's checking account *to* the investment, it will be a negative amount, and conversely, if the periodic payment goes *from* the investment *to* the checking account, it will be a positive amount.

Example

Brian Quille would like to buy a sailboat for $30,000 in 7 years. He expects to earn 10% compounded quarterly on his investments. What amount should Brian invest each quarter to attain his goal?

Elements:

Present Value	N/A
Future Value	30000
Payments	solve for
Number of periods	7 × 4
Interest	10 (÷ 4 for 12C)

$30,000 is the future value because it is the amount that will be achieved that the end of the investment timeline.

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
4, gold, P/YR	---
30000, FV	30000, FV
7, gold, ×P/YR	7, ENTER, 4, ×, n
10, I/YR	10, ENTER, 4, ÷, i
PMT	PMT

Answer: $753

Notice that the answer that appears on your calculator will be negative. This is because Brian will need to move money from his checking account to fund the investment.

For problems where the payments are made toward a loan or other debt, the amount loaned is the present value (PV).

Example

Edie Mann borrowed $17,320 for a new car. The loan has a 3-year term, and the interest rate is 5.9% Calculate Edie's monthly payment.

Elements:

Present Value	17320
Future Value	N/A
Payments	solve for
Number of periods	3 × 12
Interest	5.9 (÷ 12 for 12C)

$17,320 is the present value because it is the amount that is loaned at the beginning of the timeline. It is a positive value because the money flowed to Edie so she could purchase the car (although it bypassed his checking account and went directly to the car dealership, the same principle applies).

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
12, gold, P/YR	---
17320, PV	17320, PV
3, gold, ×P/YR	3, ENTER, 12, ×, n
5.9, I/YR	5.9, ENTER, 12, ÷, i
PMT	PMT

Answer: $526

Again, the answer that appears on your calculator will be negative. This is because Edie will need to move money from her checking account for the monthly loan payments.

Practice Problems

1. Billie Frisbie invested her inheritance of $250,000 into an investment paying 6% quarterly. She would like to withdraw equal periodic payments at the end of each quarter for the next 18 years. How much can she receive each quarter?

Elements:

Present Value	–250000
Future Value	N/A
Payments	solve for
Number of periods	18 × 4
Interest	6 (÷ 4 for 12C)

Keystrokes:

10B	12C
gold, clear all	f, CLX
4, gold, P/YR	---
–250000, PV	–250000, PV
18, gold, ×P/YR	18, ENTER, 4, ×, n
6, I/YR	6, ENTER, 4, ÷, i
PMT	PMT

Answer: $5,702

2. Chandra Singh wants to accumulate $450,000 for retirement at age 65. She is 27 today and has decided to make semi-annual payments into a money market account paying 8.2% What amount should Chandra's semi-annual payments be?

Elements:

Present Value	N/A
Future Value	450000
Payments	solve for
Number of periods	38 × 2
Interest	8.2 (÷ 2 for 12C)

Keystrokes:

10B	12C
gold, clear all	f, CLX
2, gold, P/YR	---
450000, FV	450000, FV
38, gold, ×P/YR	38, ENTER, 2, ×, n
8.2, I/YR	8.2, ENTER, 2, ÷, i
PMT	PMT

Answer: $914

3. Carmen Reyes has just retired. She has received $175,000 (after-taxes) in a lump-sum from her company pension plan. She would like to receive equal periodic payments at the end of every month for the next 20 years. She assumes she can earn 8.25% on her investments. What monthly amount can Carmen expect to receive?

Elements:

Present Value	–175000
Future Value	N/A
Payments	solve for
Number of periods	20 × 12
Interest	8.25 (÷ 12 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
12, gold, P/YR	---
–175000, PV	–175000, PV
20, gold, ×P/YR	20, ENTER, 12, ×, n
8.25, I/YR	8.25, ENTER, 12, ÷, i
PMT	PMT

Answer: $1,491

4. Carol and Alan Hapman want to purchase a home in 6 years. They will need $125,000. To reach this goal, what amount should the Hapmans invest at the end of every 6-month period if they expect to earn a 9% rate of return, compounded semi-annually on their investment?

Elements:

Present Value	N/A
Future Value	125000
Payments	solve for
Number of periods	6 × 2
Interest	9 (÷ 2 for 12C)

Keystrokes:

10B	12C
gold, clear all	f, CLX
2, gold, P/YR	---
125000, FV	125000, FV
6, gold, ×P/YR	6, ENTER, 2, ×, n
9, I/YR	9, ENTER, 2, ÷, i
PMT	PMT

Answer: $8,083

Adjusting for inflation

When investing for a future goal, inflation erodes the value of invested funds. There is a formula that allows you to adjust the interest rate for inflation also known as the real rate of return. Whenever a question asks for the result today's dollars, use the real rate return formula as follows:

$$\text{Real Rate of Return} = \left[\frac{1+r}{1+i} - 1\right] \times 100$$

r represents the after-tax rate of return and **i** represents the rate of inflation, both expressed as decimals

Example

Your client expects to make 8% on her investments and inflation is predicted to average 3%. What is your client's inflation-adjusted rate of return?

$$\text{Real Rate of Return} = \left[\frac{1+.08}{1+.03} - 1\right] \times 100$$

Keystrokes:

10B	12C
1.08, ÷, 1.03, =, -1, =, ×, 100, =	1.08, ENTER, 1.03, ÷, 1, - 100, ×, i

Answer: 4.85%

Payments made or received at the beginning of a period

You may remember that your calculator can be set to calculate payments made at the *beginning* of a period (up to this point, all our calculations have used the default End mode for payments).

To toggle between Begin and End on the **10B**, press the **gold** key followed by the **Beg/End** key (the key with **MAR** in white). BEG will appear on the display in Begin mode and disappear in End mode. **Always check to ensure that your calculator is in the correct mode before beginning a problem involving payments.**

To change to Begin mode on the **12C**, press **g** followed by **BEG** (the key with **7** in white). BEGIN will appear on the display. To change to End mode, press **g** followed by **END** (the key with **8** in white). BEGIN will disappear when the calculator is in End mode.

The following example uses the real rate of return as the interest rate as well as payments received at the beginning of each period.

Example

Your client wants to receive $40,000 in terms of today's dollars at the beginning of each year for the next five years. He read that inflation will average 5% for the next decade. His intention is to use an investment vehicle providing a reliable average return of 9%. This client wishes to invest a lump sum today. The client also understands that all the money will be used up at the beginning of the fifth year.

Elements:

Present Value	solve for
Future Value	N/A
Payments	40000 (Begin mode)
Number of periods	5
Interest	$\left[\frac{1+.09}{1+.05}-1\right]\times 100$

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
gold, Beg/End (display shows BEG)	g, BEG
1, gold, P/YR	---
40000, PMT	40000, PMT

5, N	5, n
1.09, ÷, 1.05, =, –1, =, ×, 100, =, I/YR	1.09, ENTER, 1.05, ÷, 1, –, 100, ×, i
PV	PV

Answer: $185,850

Practice Problems

1. Henry Morello needs an income stream equivalent to $32,000 in today's dollars at the beginning of each year for the next 9 years to maintain his standard of living. He assumes inflation will average 5% over the long run and he can earn a 10% compound annual after-tax rate of return on investments. What lump-sum does Henry need to invest today to fund his needs?

Elements:

Present Value	solve for
Future Value	N/A
Payments	32000 (Begin mode)
Number of periods	9
Interest	$\left[\frac{1+.10}{1+.05}-1\right]\times 100$

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
gold, Beg/End (display shows BEG)	g, BEG
1, gold, P/YR	---
32000, PMT	32000, PMT
9, N	9, n
1.1, ÷, 1.05, =, –1, =, ×, 100, =, I/YR	1.1, ENTER, 1.05, ÷, 1, –, 100, ×, i
PV	PV

Answer: $240,828

2. Lisa Wiley wants to retire in 12 years. She needs an additional $390,000 to sufficiently fund this objective. She assumes inflation will run 6% over the long-run and she can earn a 7% compound annual after-tax return on her investments. What will her annual payments be?

Elements:

Present Value	N/A
Future Value	390000
Payments	solve for (End mode)
Number of periods	12
Interest	$\left[\frac{1+.07}{1+.06}-1\right]\times 100$

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
gold, Beg/End (display clears)	g, END
390000, FV	390000, FV
12, N	12, n
1.07, ÷, 1.06, =, –1, =, ×, 100, =, I/YR	1.07, ENTER, 1.06, ÷, 1, –, 100, ×, i
PMT	PMT

Answer: PMT $30,848

3. Arthur and Jane Perry are ready to retire. They want to receive $86,000 at the beginning of each year for the next 20 years. They assume inflation will run 3% over the long run and they can earn a 12% compound annual after-tax rate of return on their investments. How much do the Perrys need to deposit today to reach their goal?

Elements:

Present Value	solve for
Future Value	N/A
Payments	86000 (Begin mode)
Number of periods	20
Interest	$\left[\frac{1+.12}{1+.03}-1\right]\times 100$

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
gold, Beg/End (display shows BEG)	g, BEG
1, gold, P/YR	---
86000, PMT	86000, PMT
20, N	20, n
1.12, ÷, 1.03, =, –1, =, ×, 100, =, I/YR	1.12, ENTER, 1.03, ÷, 1, –, 100, ×, i
PV	PV

Answer: $869,841

Bond calculations

You may be asked to solve problems related to bond yields. You can apply the principles of time value of money to solve problems involving bonds.

Determining a Bond's Current Price

If you know the intrinsic price of a bond, you can determine if a securities broker's mark-up on a bond is reasonable or not.

Elements:

Present Value	solve for
Future Value	par/face value
Payments	interest rate × par value ÷ payments per year
Number of periods	number of payment periods
Interest	coupon rate ÷ number of payments per year

Note: Bonds often have a par value of $1,000 and pay out semi-annually, but not all problems assume this, so be sure to read carefully.

Example

Kevin O'Brien owns a KAf, Inc. corporate bond with a face value of $1,000 and a coupon rate of 7% payable semi-annually. This bond matures in 15 years. Comparable bonds with the same maturity are yielding 8%. Use this information to determine the intrinsic price of Kevin's bond.

Elements:

Present Value	solve for
Future Value	1000
Payments	35 (7% of 1000 ÷ 2)
Number of periods	30 (15 × 2)
Interest	8 (÷ 2 for 12C)

Keystrokes:

10B	12C
gold, clear all	f, CLX
2, gold, P/YR	---
1000, FV	1000, FV
15, gold, ×P/YR	30, n
8, I/YR	8, ENTER, 2, ÷, i
35, PMT	35, PMT
PV	PV

Answer: $913.54

Practice Problems

1. Frank Sorich is considering buying a Maple County Housing Authority bond with a coupon rate of 7.5% semi-annually. The bond matures in 5 years. Comparable bonds are yielding 8%. What price should Frank be willing to pay for this bond?

Elements:

Present Value	solve for
Future Value	1000
Payments	37.5 (7.5% of 1000 ÷ 2)
Number of periods	10 (5 × 2)
Interest	8 (÷ 2 for 12C)

Keystrokes:

10B	12C
gold, clear all	f, CLX
2, gold, P/YR	---
1000, FV	1000, FV
37.5, PMT	37.5, PMT

5, gold, ×P/YR	10, n
8, I/YR	8, ENTER, 2, ÷, i
PV	PV

Answer: $980

2. Vidal Lora owns a CPQ, Inc. corporate bond with a coupon rate of 6%, payable semi-annually. The bond matures in 8 years and comparable bonds are yielding 5%. If Vidal were to sell this bond, what price should he receive?

Elements:

Present Value	solve for
Future Value	1000
Payments	30 (6% of 1000 ÷ 2)
Number of periods	16 (8 × 2)
Interest	5 (÷ 2 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
2, gold, P/YR	---
1000, FV	1000, FV
30, PMT	30, PMT
8, gold, ×P/YR	16, n
5, I/YR	5, ENTER, 2, ÷, i
PV	PV

Answer: $1,065

Determining a Bond's Yield to Maturity

Yield to maturity is the total return on a bond when held to maturity. In yield to maturity problems, we are calculating the yield (interest rate) when we know the present value (what the bond cost the investor, not the face value), future value (face value), payments (coupon rate × face value ÷ # of payments per year), and number of periods remaining (# of years × # of payments per year).

Example

Sophie Wolinsky bought a Cooke Industry bond with a coupon rate of 8.5% payable semi-annually. Sophie paid $935 to acquire this bond which matures in 9 years. What is the yield to maturity on Sophie's bond?

Elements:

Present Value	-935
Future Value	1000
Payments	42.5 (8.5% of 1000 ÷ 2)
Number of periods	18 (9 × 2)
Interest	solve for (× 2 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
2, gold, P/YR	---
935, +/–, PV	935, CHS, PV
1000, FV	1000, FV
42.5, PMT	42.5, PMT
9, gold, ×P/YR	18, n
I/YR	i, ENTER, 2, ×

Answer: 9.59%

Determining a Bond's Yield to Call

Calculating the yield to call for bonds which are called involves a similar process to calculating yield to maturity; however, when a bond is called several of the problem's elements change. When a bond is called away from its holder (the investor), the issuer usually provides that unhappy holder with a small redemption bonus which is most often referred to as a call penalty (to the issuer) or a call premium (to the investor). In a bond which is called, do **not** assume a $1,000 future value (FV). The future value will be the redemption price plus the call premium. If the call premium is $90.00, the future value (FV) of that called bond will be $1,090.

The number of periods (N/n) will also change. Although a problem will provide information about a bond's years remaining to maturity, this information is not relevant to an investor who must turn in their bonds before they mature. When determining the number of periods (N/n) element in a yield to call problem, use the number of years to call × the number of payments.

Example

Lee Park bought a callable 10-year Megabiz, Inc. Bond with a coupon rate of 9% payable semi-annually. Lee paid $980 for the bond which was called in 5 years and provided a call premium of $85.00. What was the yield to call on this Megabiz bond?

Elements:

Present Value	-980
Future Value	1085
Payments	45 (9% of 1000 ÷ 2)
Number of periods	10 (5 × 2)
Interest	solve for (× 2 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
2, gold, P/YR	---
980, +/–, PV	980, CHS, PV
1085, FV	1085, FV
45, PMT	45, PMT
5, gold, ×P/YR	10, n
I/YR	i, ENTER, 2, ×

Answer: 10.85%

Practice Problems

1. Charles Nshimo purchased a bond for $1,225. The coupon rate is 10%, payable semi-annually. The bond is callable in 5 years at $1,080 and the bond matures in 15 years. What is the yield to maturity on this bond?

Elements:

Present Value	-1225
Future Value	1000
Payments	50 (10% of 1000 ÷ 2)
Number of periods	30 (15 × 2)
Interest	solve for (× 2 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
2, gold, P/YR	---
1225, +/–, PV	1225, CHS, PV
1000, FV	1000, FV
50, PMT	50, PMT
15, gold, ×P/YR	30, n
I/YR	i, ENTER, 2, ×

Answer: 7.48%. The question asks for yield to maturity; therefore, call information is irrelevant.

2. What is the yield to call on the bond in the previous problem?

Elements:

Present Value	-1225
Future Value	1080
Payments	50 (10% of 1000 ÷ 2)
Number of periods	10 (5 × 2)
Interest	solve for (x 2 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
2, gold, P/YR	---
1225, +/–, PV	1225, CHS, PV
1080, FV	1080, FV
50, PMT	50, PMT
5, gold, ×P/YR	10, n
I/YR	i, ENTER, 2, ×

Answer: 6.10%

3. Hazel Kelber purchased a bond for $790. The coupon rate is 9%, payable semi-annually and the bond is callable in 10 years at $1,150. The bond matures in 20 years. What is the yield to call on this bond?

Elements:

Present Value	-790
Future Value	1150
Payments	45 (9% of 1000 ÷ 2)
Number of periods	20 (10 × 2)
Interest	solve for (× 2 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
2, gold, P/YR	---
790, +/–, PV	790, CHS, PV
1150, FV	1150, FV
45, PMT	45, PMT
10, gold, ×P/YR	20, n
I/YR	i, ENTER, 2, ×

Answer: 13.66%

Three Step TMV Calculations (College/Retirement)

The CFP® Exam is likely to include three-step calculations in the context of college education funding and retirement income needs analysis. While at first, these problems may feel intimidating, once they are broken down into the appropriate steps they are easily managed. The rates that are used in the calculations are different in the various steps and may reflect inflation, investments, and inflation-adjusted rates of return.

Step 1: Inflating the required amount for the period until it is needed

If annual needs are presented in "today's dollars," "today's money," or "adjusted for inflation," the first step is to inflate today's dollars into what they will become at the point when the money will ultimately be needed.

This is actually a time value of money problem in which you are solving for future value (the future amount you will need because of the effect of inflation) with the following elements:

Present Value	The amount needed in today's dollars (negative value)
Future Value	solve for
Payments	N/A
Number of periods	The number of years until the money is needed
Interest	The rate of inflation*

*The inflation rate may be provided explicitly or referred to as CPI (Consumer Price Index). For education funding problems, you may be provided with the rate that college tuition is increasing.

Note: Not all retirement and education planning problems require this step. If the question doesn't mention "today's dollars," "today's money," or "adjusted for inflation," you do not need to complete Step 1.

Step 2: Calculating the lump sum needed for retirement or college funding

In this step, we are calculating the lump-sum amount that will be needed when college or retirement begins. In most problems, retirement costs or college payments start on day one and are considered payments to the retiree or to the college; therefore, your calculator will almost always be in **begin mode**. This is a time value of money problem in which you are solving for present value (the value you need at the beginning of the period given how much is needed per year). The elements are as follows:

Present Value	solve for
Future Value	0
Payments	The amount needed at the beginning of each year (begin mode), if you did Step 1 for the problem, this will be the FV answer that you calculated
Number of periods	The number of years the money will be needed, i.e., the number of years of retirement or college
Interest	The real rate of return of your investment $\left[\frac{1+r}{1+i} - 1\right] \times 100$

You will recognize the real rate of return formula from the *Adjusting for Inflation* section above. It may be tempting to simply subtract the inflation rate from the after-tax investment rate. Don't fall into this trap! There is a strong possibility that one of the wrong answers reflects this misguided shortcut.

Note: Certain retirement needs analysis or college funding questions may only require two steps and specifically ask for the lump-sum amount that is needed when college or retirement begins.

Step 3: Calculating required funding

This step calculates the amount of funding in dollars that is needed for the goal. The question may ask for one lump-sum to fund the goal, annual payments, or monthly payments.

The elements will be as follows:

Present Value	Solve for if the problem asks for the amount required at the beginning of the funding period. If starting from zero, do not enter a PV. If giving a starting amount, use that amount as the PV.
Future Value	The amount calculated in Step 2. It's possible that in one-step problems, the amount required will be provided and there will be no need to complete steps 1 and 2.
Payments	Solve for if the problem asks for the annual or monthly savings would need to be
Number of periods	The number of years or months before the funding is needed (based on the compounding period provided in the problem).
Interest	Rate that the investment will grow

Important points when calculating required funding:

- Use the rate of return provided. Do not calculate the inflation adjusted rate of return because you already adjusted for inflation.
- Unless the question specifically indicates that the client will make retirement savings payments at the beginning of each year or month, presume that payments are made at the **end** of the interval.

Example

Mr. and Mrs. Happycouple, age 45 and 46 respectively intend to retire in 22 years. It has been determined that they will need $125,000 annually in today's dollars in their estimated 30 years of retirement. As moderate-risk tolerance investors, they anticipate an after-tax rate of return of 7%. Due to recent inflationary spikes, they estimate that inflation will average 4% in the foreseeable future. To fund this retirement need, what amount should Mr. and Mrs. Happycouple save at the end of each month to achieve their retirement income needs goal?

Step 1:

Elements:

Present Value	–125000
Future Value	solve for
Payments	N/A
Number of periods	22
Interest	4

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
125000, +/–, PV	125000, CHS, PV
22, N	22, n
4, I/YR	4, i
FV	FV

Answer: $296,240

Step 2:

Elements:

Present Value	solve for
Future Value	N/A
Payments	296240 (FV from Step 1—use **begin mode**)
Number of periods	30
Interest	$\left[\frac{1+.07}{1+.04}-1\right]\times 100$

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
gold, Beg/End (display shows BEG)	g, BEG
296240, +/–, PMT	296240, CHS, PMT
1.07, ÷, 1.04, =, –, 1, =, ×, 100, =, I/YR	1.07, ENTER, 1.04, ÷, 1, –, 100, ×, i
30, N	30, n
PV	PV

Answer: $6,064,022

Step 3:

Elements:

Present Value	N/A or 0
Future Value	6064022
Payments	solve for
Number of periods	264 (22 × 12)
Interest	7 (÷ 12 for 12C)

Keystrokes:

10B	12C
gold, clear all	f, CLX
12, gold, P/YR	---
gold, Beg/End (display clears)	g, END
6064022, FV	6064022, FV
22, gold, ×P/YR	264, n
7, I/YR	7, ENTER, 12, ÷, i
PMT	PMT

Answer: $9,708

Practice Problems

1. Savannah Saver currently earns $90,000 per year as an independent interior designer. She is currently age 45. She expects that she will spend 80% of her current earnings in her post-retirement years which will begin when she reaches age 65. She expects that her Social Security retirement benefit will amount to $2,500 per month. As an investor with a relatively high-risk tolerance, Savannah expects to earn an after-tax rate of 10% on her investments. She and her financial planner have agreed to assume that inflation will average 3% in the future. She expects to live to age 85. Savanna is a disciplined saver who will make deposits at the end of each month to achieve the goal of adequate retirement income.

Step 1:

Elements:

Present Value	–42000 (80% of 90,000 = 72,000 less 30,000 per year Social Security)
Future Value	solve for
Payments	N/A
Number of periods	20
Interest	3

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
42000, +/–, PV	42000, CHS, PV
20, N	20, n
3, I/YR	3, i
FV	FV

Answer: $75,857

Step 2:
Elements:

Present Value	solve for
Future Value	N/A
Payments	75857 (FV from Step 1—use **begin mode**)
Number of periods	20
Interest	$\left[\frac{1+.1}{1+.03}-1\right]\times 100$

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
gold, Beg/End (display shows BEG)	g, BEG
75857, +/–, PMT	75857, CHS, PMT
1.1, ÷, 1.03, =, –, 1, =, ×, 100, =, I/YR	1.1, ENTER, 1.03, ÷, 1, –, 100, ×, i
20, N	20, n
PV	PV

Answer: $872,016

Step 3:

Elements:

Present Value	N/A
Future Value	872016
Payments	solve for
Number of periods	240 (20 × 12)
Interest	10 (÷ 12 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
12, gold, P/YR	---
gold, Beg/End (display clears)	g, END
872016, FV	872016, FV
20, gold, ×P/YR	240, n
10, I/YR	10, ENTER, 12, ÷, i
PMT	PMT

Answer: $1,148

2. It has been determined that when Matthew Mathmajor enters college sixteen years from now his parents need to have accumulated $485,000 by that time. Presume that the consumer price index is 6% and the college inflation rate is 8%. Matthew's parents are somewhat conservative investors who expect an after-tax rate of return of 5% on their selected investments. What amount of money should Matthew's parents deposit at the end of each month to meet this goal?

Note: This problem is asking only what monthly investment needs to be made to accumulate the target amount ($485,000); therefore, it is a "Step 3" problem only. The information about CPI and college inflation is superfluous and meant to trick you into thinking you need to complete more steps. Your calculator should be in End mode for this problem.

Elements:

Present Value	N/A
Future Value	485000
Payments	solve for
Number of periods	192 (16 × 12)
Interest	5 (÷ 12 for 12C)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
12, gold, P/YR	---
485000, FV	485000, FV
16, gold, ×P/YR	192, n
5, I/YR	5, ENTER, 12, ÷, i
PMT	PMT

Answer: $1,654

Internal Rate of Return and Net Present Value calculations

Understanding Internal Rate of Return

An investment's internal rate of return (IRR) represents the true annual rate of earnings on an investment. IRR takes into account the gain or loss on an investment, the cashflow (positive or negative) which is produced by an investment, and the amount of time between an investment's purchase and sale (the holding period) and creates a rate of total return on an annualized basis. The internal rate of return (IRR) is often referred to as **total return**.

Determining the Internal Rate of Return for investments with no periodic cashflows

Many investments come with no cashflow. There is neither interest nor dividends nor net rentals. These are buy-hold-and-sell arrangements such as zero-coupon bonds and single purchases of physical assets. Solving for the IRR on an investment with no periodic payments requires a simple format: you can simply solve for the interest (i or I/YR) element as you have been in prior time value of money problems that ask you to solve for the rate of return.

Calculating Internal Rate of Return for investments with equal periodic payments

The previous section on bond calculations, we calculated a bond's yield to maturity and yield to call. Many fixed income investments have regular periodic payments which are equal in amount. Most bonds fall into this category as does preferred stock. Most lease investments return fixed rental payments. When payments remain regular in terms of both interval and amount, the PMT key can be used to input the calculator as to the cashflow element. Therefore, these problems are actually internal rate of return calculations in which the periodic payments are equal.

Example

Pablo Esposito acquired an Ideas, Incorporated Senior Mortgage Bond with a 9.5% coupon rate payable semi-annually. The bond will mature in 12 years. Pablo acquired the bond for a price of $1,175 in the secondary market. If Pablo holds this bond to maturity, what will his internal rate of return be?

Elements:

Present Value	–1175
Future Value	1000
Payments	47.5 (9.5% of 1000 ÷ 2)
Number of periods	24 (12 × 2)
Interest	solve for (× 2 for 12C)

The future value of the bond at maturity is $1,000, because that is the standard face or par value of a corporate bond.

Keystrokes:

10B	12C
gold, clear all	f, CLX
2, gold, P/YR	---
1175, +/–, PV	1175, CHS, PV
1000, FV	1000, FV
47.5, PMT	47.5, PMT
12, gold, ×P/YR	24, n
I/YR	i, ENTER, 2, ×

Answer: 7.29%

Practice Problems

1. Rory Macon purchased a U.S. Treasury bond with a coupon rate of 8.5%, paid semi-annually. The bond will mature in 5 years. He bought the bond in the secondary market for $1,010. What is the internal rate of return on this investment?

Elements:

Present Value	–1010
Future Value	1000
Payments	42.5 (8.5% of 1000 ÷ 2)
Number of periods	10 (5 × 2)
Interest	solve for (× 2 for 12C)

Keystrokes:

10B	12C
gold, clear all	f, CLX
2, gold, P/YR	---
1010, +/–, PV	1010, CHS, PV
1000, FV	1000, FV

42.5, PMT	42.5, PMT
5, gold, ×P/YR	10, n
I/YR	i, ENTER, 2, ×

Answer: 8.25%

2. Florence Grace bought 150 shares of PPP, Inc. Preferred stock for $25.00 per share. It paid her a dividend of $56.25 every quarter. She held the stock 2.5 years before selling it at $24.00 per share. What was Florence's IRR on this stock?

Elements:

Present Value	–25
Future Value	24
Payments	56.25 ÷ 150*
Number of periods	2.5 × 4
Interest	solve for (× 4 for 12C)

*Or, alternatively, you could multiply the present and future share prices times the number of shares and use $56.25 as the payment amount. The answer will be the same.

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
4, gold, P/YR	---
25, +/–, PV	25, CHS, PV
24, FV	24, FV
56.25, ÷, 150, =, PMT	56.25, ENTER, 150, ÷, PMT
2.5, gold, ×P/YR	2.5, ENTER, 4, ×, n
I/YR	i, ENTER, 4, ×

Answer: 4.48%

3. Bertram Blaine purchased 10 units of a tax-free unit investment trust to fund his retirement. He bought each unit for $892 and has received $118.33 every month for 6 years. He sold his UIT for $1,002 per unit at the end of the sixth year. What was Bertram's internal rate of return on this investment?

Elements:

Present Value	–892 × 10
Future Value	1002 × 10
Payments	118.33*
Number of periods	6 × 12
Interest	solve for (× 12 for 12C)

*Or, alternatively, you could divide the payment amount by the number of units and use the unit price as the present and future values, whichever makes more sense to you. Again, the answer will be the same either way.

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
12, gold, P/YR	---
892, ×, 10, =, +/–, PV	892, ENTER, 10, ×, CHS, PV
1002, ×, 10, =, FV	1002, ENTER, 10, ×, FV
118.33, PMT	118.33, PMT
6, gold, ×P/YR	6, ENTER, 12, ×, n
I/YR	i, ENTER, 12, ×

Answer: 17.11%

4. Natalie Rokowski invested $20,000 in newly issued 5-year Treasury notes 4 years ago. She received payments of 10.25% semi-annually. The bonds are worth $26,215 today. What is the IRR on this investment?

Elements:

Present Value	–20000
Future Value	26215
Payments	1025*
Number of periods	4 × 2
Interest	solve for (× 2 for 12C)

*10.25% of $20,000 is $2,050 divided by 2 payments per year is $1,025

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
2, gold, P/YR	---
20000, +/–, PV	20000, CHS, PV
26215, FV	26215, FV
1025, PMT	1025, PMT
4, gold, ×P/YR	4, ENTER, 2, ×, n
I/YR	i, ENTER, 2, ×

Answer: 16.08%

5. Ben Abraham invested $10,000 in HBC, Inc. Preferred stock. He was paid quarterly dividends of 6.5%. Six years later, he sold all of his shares of HBC for $9,265. What was his internal rate of return on this investment?

Elements:

Present Value	–10000
Future Value	9265
Payments	650 ÷ 4
Number of periods	6 × 4
Interest	solve for (× 4 for 12C)

Keystrokes:

10B	12C
gold, clear all	f, CLX
4, gold, P/YR	---
10000, +/–, PV	10000, CHS, PV
9265, FV	9265, FV
650, ÷, 4, =, PMT	650, ENTER, 4, ÷, PMT
6, gold, ×P/YR	6, ENTER, 4, ×, n
I/YR	i, ENTER, 4, ×

Answer: 5.46%

Calculating Internal Rate of Return for Investments with Unequal Payments

Many types of investments provide uneven amounts of cashflow or cashflow payments during some periodic intervals, but not others. Stocks exemplify unequal payment investments most often because many investors hold them in their portfolios. Remember, stocks pay dividends only if they are earned and distributed. The amount of dividends can vary greatly. Mutual funds are likely to have uneven payments as are many types of limited partnerships. Multiple purchases of physical assets also fall into this category.

Because cashflow distributions (payments) are not even and regular, a PMT element will not work as an element in unequal payment IRR problems. Instead, a chronological data entry sequence must be followed. This method is known as the Cash Flow (Entry) Method. This method will require the use of the keys we have not used previously in the manual.

Function	Description	10B	12C
Original Cashflow	The initial deposit which begins an investment. Because it is an *investment*, the value is always entered as *negative*. The 10B does not distinguish between the initial deposit and subsequent (known as juxtaposed) cashflows, so the same key is used for all.	CFj	g, CFo (the key with PV in white)

Additional Cashflows	The cashflow immediately following the previous cashflow. If the cashflow is an investment, it is expressed as a negative value. If the cashflow is income (e.g., rent), it is expressed as a positive value. If there is neither investment nor income for the period, it should still be entered with the value zero.	CFj	g, CFj (the key with PMT in white)
Internal Rate of Return	The total return of an investment on an annualized basis. IRR is expressed as a percentage. The 10B calculates IRR per year, so 10B users will need to set periods per year to 1 before beginning IRR problems.	gold, IRR/YR (the key with **CST** in white)	f, IRR (the key with **FV** in white)

It is important to write down the cashflows in order before entering in your calculator so that you are sure not to miss a cashflow.

Example

Three years ago, Cara Vitelli bought a rare first edition book for $2,500. She added to her collection by buying another first edition for $1,500. The second year, there were no attractive buys, so her investment was zero. The third year, Cara bought another first edition book for $2,250. The whole collection was sold in the third year for $10,000. What was Cara's IRR on this investment?

Elements:

Original investment	–2500
year 1	–1500
year 2	0
year 3	7750 (–2250 + 10000)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---

2500, +/–, CFj	2500, CHS, g, CFo
1500, +/–, CFj	1500, CHS, g, CFj
0, CFj	0, g, CFj
7750, CFj	7750, g, CFj
IRR (gold CST) key	IRR (gold FV) key

Answer: 28.31%

If you neglected to enter zero for year 2, your answer would be wildly wrong because IRR would have been calculated for 2 years instead of 3. 10B users who forget to reset periods per year to 1 would also get completely wrong answers.

Correctly determining the IRR on a real estate investment involves factoring the effect of the mortgage into the equation. Remember that the original cashflow (CFo or CFj for 10B users) is the *down payment* and not the purchase price. IRR is factored based on the *investor s* money in and out of the deal. In real estate, the investor's money in the deal is the down payment.

The final year's cashflow encompasses three events:
1. the sales proceeds
2. the return of the interest-only mortgage amount or mortgage balance to the lender
3. the net operational inflow or outflow.

Remember to combine these three events into a single dollar amount.

Example

Arnold McCormick bought an 8-unit apartment building for $250,000 with a down payment of $50,000 using an interest-only mortgage. Some renovation was required during the first year; therefore, the first year's net cashflow was a negative -$16,000. The following years proved to be profitable with positive net cashflow of $9,000 the second year, $18,000 the third year, and $20,000 the fourth year. Arnold sold the building at the end of the fourth year for $580,000. What was Mr. McCormick's internal rate of return on this investment?

Elements:

Original investment	–50000 (down payment)
year 1	–16000
year 2	9000
year 3	18000
year 4	580000 (proceeds of sale) – 200000 (mortgage amount) + 20000 (final year proceeds) = 400000

Keystrokes:

10B	12C
gold, clear all	f, CLX
1, gold, P/YR	---
50000, +/–, CFj	50000, CHS, g, CFo
16000, +/–, CFj	16000, CHS, g, CFj
9000, CFj	9000, g, CFj
18000, CFj	18000, g, CFj
400000, CFj	400000 g, CFj
IRR (gold CST) key	f, IRR

Answer: 66.18%

Practice Problems

1. Stanley Sewak bought 100 shares of GHB Corp. at $17 per share. He held the stock for 4 years and sold it at $25 per share. In years 1 and 3, he received annual dividends of $125 and $160, respectively. What was the IRR on this investment?

Elements:

Original investment	–1700 (17 × 100)
year 1	125
year 2	0
year 3	160
year 4	2500 (25 × 100)

Keystrokes:

10B	12C
gold, clear all	f, CLX
1, gold, P/YR	---
1700, +/–, CFj	1700, CHS, g, CFo
125, CFj	125, g, CFj

0, CFj	0, g, CFj
160, CFj	160, g, CFj
2500, CFj	2500, g, CFj
IRR (gold CST) key	f, IRR

Answer: 13.96%.

2. Madelyn Botello bought a piece of Depression glass 3 years ago while she was on an antique buying junket for $125. The next year, she found another piece which she bought for $216. The year after that, she once again added to her collection by buying a piece for $192. She then posted a picture of all three pieces on social media and another collector offered her $895 for the collection. Madelyn immediately jumped at the offer. What was Madelyn's IRR on this investment?

Elements:

Original investment	–125
year 1	–216
year 2	703 (895 – 192)

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
125, +/–, CFj	125, CHS, g, CFo
216, +/–, CFj	216, CHS, g, CFj
703, CFj	703, g, CFj
IRR (gold CST) key	f, IRR

Answer: 66%

3. Hank Zanarini is considering buying an apartment building for $650,000 with a down payment of $70,000 and obtaining an interest-only loan for 5 years at which time he expects to sell the building for $810,000. He expects net annual proceeds of $2,500 the first year, $3,000 the second year, $4,000 the third year, $4,000 the fourth, and $4,600 the fifth year. What would the internal rate of return be on Hank's investment?

Elements:

Original investment	–70000
year 1	2500
year 2	3000
year 3	4000
year 4	4000
year 5	810000 (proceeds of sale) – 580000 (mortgage amount) + 4600 (final year proceeds) = 234600

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
70000, +/–, CFj	70000, CHS, g, CFo
2500, CFj	2500, g, CFj
3000, CFj	3000, g, CFj
4000, CFj	4000, g, CFj
4000, CFj	4000, g, CFj
234600, CFj	234600, g, CFj
IRR (gold CST) key	f, IRR

Answer: 30.0%

4. Gary Raia invested $7,000 in a mutual fund. A year later, he invested another $1,500. The following year, he invested $2,500. The year after that, he invested no money into the mutual fund. The year after that he invested $3,000. The next year he sold the fund and received $25,000. What was Gary's internal rate of return on his investment?

Elements:

Original investment	–7000
year 1	–1500
year 2	–2500
year 3	0
year 4	–3000
year 5	25000

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
7000, +/–, CFj	7000, CHS, g, CFo
1500, +/–, CFj	1500, CHS, g, CFj
2500, +/–, CFj	2500, CHS, g, CFj
0, CFj	0, g, CFj
3000, +/–, CFj	3000, CHS, g, CFj
25000, CFj	25000, g, CFj
IRR (gold CST) key	f, IRR

Answer 16.23%

Understanding and Calculating Net Present Value

When planners and investors wish to compare several investments, they can determine the present value of each of the investments and compare them. Thus far we have been able to determine the present value of investments with no cashflow and equal periodic. It can be tricky to calculate the present value of an investment with unequal cashflows. To do this, you will solve for net present value (NPV). A net present value (NPV) problem asks, for the price that will return a specific rate of return on an investment with varying cashflow. To calculate NPV on the 10B, press **gold** followed by the **NPV** key (the key with **PRC** in white). To calculate NPV on the 12C, press **f** followed by the **NPV** key (the key with **PV** in white).

Example

Skipper expects his investment to provide the following cash flows at the end of each of the years below:

Year 1:	150
Year 2:	–75
Year 3:	–75
Year 4:	0
Year 5:	400

Skipper's required rate of return is 9%. What is the net present value of Skipper's investment?

Elements:

Original investment	0 (not yet known)
year 1	150
year 2	–75
year 3	–75
year 4	0
year 5	400
required rate of return	9

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
0, CFj	0, g, CFo
150, CFj	150, g, CFj
75, +/–, CFj	75, CHS, g, CFj
75, +/–, CFj	75, CHS, g, CFj
0, CFj	0, g, CFj
400, CFj	400, g, CFj
9, I/YR	9, i
NPV (gold PRC) key	f, NPV

Answer: $276.55

Practice Problems

1. James Jonas has an investment that he expects to provide the following cashflows at the end of each year: Year 1: +$200, Year 2: –$50, Year 3: +$100, Year 4: –$75. James' expected rate of return is 8%. What is the NPV (net present value) of James' investment?

Elements:

Original investment	0 (not yet known)
year 1	200
year 2	–50
year 3	100
year 4	–75
required rate of return	8

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
0, CFj	0, g, CFo
200, CFj	200, g, CFj
50, +/–, CFj	50, CHS, g, CFj
100, CFj	100, g, CFj
75, +/–, CFj	75, CHS, g, CFj
8, I/YR	8, i
NPV (gold PRC) key	f, NPV

Answer: $166.57

2. Deonna Sidney is considering an investment that is expected to return the following cashflows: Year 1: –$772, Year 2: +$1,600, Year 3: $0, Year 4: +$1,753. Her expected rate of return is 11%. What would Deonna's NPV be on this investment?

Elements:

Original investment	0 (not yet known)
year 1	–772
year 2	1600
year 3	0
year 4	1753
required rate of return	11

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
0, CFj	0, g, CFo
772, +/–, CFj	772, CHS, g, CFj
1600, CFj	1600, g, CFj
0, CFj	0, g, CFj
1753, CFj	1753, g, CFj
11, I/YR	11, i
NPV (gold PRC) key	f, NPV

Answer: $1,758

3. Paul Rodriguez expects his investment to provide the following cashflows at the end of each of these years: Year 1: –$100, Year 2: +$0, Year 3: +$75, Year 4: +$75, Year 5: +$25. His required rate of return is 7.5%. What is the NPV of Paul's investment?

Elements:

Original investment	0 (not yet known)
year 1	–100
year 2	0
year 3	75
year 4	75
year 5	25
required rate of return	7.5

Keystrokes:

10B	**12C**
gold, clear all	f, CLX
1, gold, P/YR	---
0, CFj	0, g, CFo
100, +/–, CFj	100, CHS, g, CFj
0, CFj	0, g, CFj
75, CFj	75, g, CFj
75, CFj	75, g, CFj
25, CFj	25, g, CFj
7.5, I/YR	7.5, i
NPV (gold PRC) key	f, NPV

Answer: $41